it was difficult to control the oscillation merely by the circuit impedance adjustment as Quate, *et al.*,[2] Makurat, *et al.*[4] and Thomas, *et al.*[6] experienced. They tried to amplify signals whose frequency was located right at the center of the strong oscillation region of repeller voltage. They loaded the tube down until it stopped oscillating and then placed an imput signal to be amplified at the original oscillation frequency. In such cases the amplifiers were often noisy.[2,4,6]

Note that in the method explained in this communication the oscillation frequency was always electrically shifted until the oscillation died out at the signal frequency and the tube was not simply overloaded to stop oscillation without shifting its frequency.

For small signals (almost the noise level of the RW-T R-BI receiver 1-μs pulse modulated, (Fig. 2, bottom) the amplifier showed a gain of 30 db at 70.35 kMc (Fig. 2, top).

The authors wish to thank S. Krupnik, Jr., J. A. Stefancin, J. E. Billo, and E. Starr for their assistance, and R. G. Rockwell of Varian Associates as well as Raytheon Company, Waltham, Mass. for contributing the klystron tubes.

K. Ishii
D. E. Schumacher
Dept. of Elec. Engrg.
Marquette University
Milwaukee, Wis.

Still Another Method for Transforming Impedances Through Lossless Networks*

The input impedance $\mathbf{Z}_A$ and the terminating impedance $\mathbf{Z}_L$ of a symmetrical lossless four-terminal network are related by an equation which contains only one additional parameter; namely, the input impedance $\mathbf{Z}_1$ when the network is terminated in any known resistance R_0. Let $\mathbf{\Gamma}_A=(\mathbf{Z}_A-R_0)/(\mathbf{Z}_A+R_0)$, $\mathbf{\Gamma}_L=(\mathbf{Z}_L-R_0)/(\mathbf{Z}_L+R_0)$, $\mathbf{\Gamma}_1=(\mathbf{Z}_1-R_0)/(\mathbf{Z}_1+R_0)$, and the superscript "*" denote the conjugate. Now

$$\mathbf{\Gamma}_A=\frac{\mathbf{\Gamma}_1}{\mathbf{\Gamma}_1{}^*}\,\frac{\mathbf{\Gamma}_1{}^*-\mathbf{\Gamma}_L}{1-\mathbf{\Gamma}_1\mathbf{\Gamma}_L}\cdot \qquad (1)$$

Eq. (1) also relates the voltage reflection coefficients for a lossless two-port junction.[1] This equation provides a basis for a graphical procedure for finding $\mathbf{Z}_A$ or $\mathbf{Z}_L$ when the other is known.

Symmetrical Networks

First, it is assumed that the network under consideration is both lossless and symmetrical. Some convenient value of R_0 is selected and the corresponding value of $\mathbf{Z}_1$ is computed or determined experimentally. (In the case of a two-port junction, R_0 would probably correspond to a matched load.) The impedance $\mathbf{Z}_{1N}=\mathbf{Z}_1/R_0$ is plotted on a Smith or Carter chart as shown in Fig. 1. The line $C\mathbf{Z}_{1N}$ is drawn from the center C of the chart through $\mathbf{Z}_{1N}$. The point M is located on this line so that $(\overline{C\mathbf{Z}_{1N}})(\overline{CM})=(\text{radius of chart})$.[2] A graphical procedure for locating M is to draw the line DE through $\mathbf{Z}_{1N}$ perpendicular to the line $C\mathbf{Z}_{1N}$. The points D and E are the intersections of this line with the $R=0$ circle. The lines DM and EM are tangent to the $R=0$ circle.

If $\mathbf{Z}_L$ is known, the impedance $\mathbf{Z}_{LN}{}^*=\mathbf{Z}_L{}^*/R_0$ is plotted. The line GM is drawn from M through $\mathbf{Z}_{LN}{}^*$. The line $\mathbf{Z}_{1N}\mathbf{Z}_{LN}{}^*$ is drawn. The line CH is drawn from C so that $\angle \mathbf{Z}_{1N}\mathbf{Z}_{LN}{}^*\mathrm{M}=\angle MCH=\theta$. The intersection of the lines MG and CH is $\mathbf{Z}_{AN}=\mathbf{Z}_A/R_0$.

If $\mathbf{Z}_A$ is known, $\mathbf{Z}_{AN}=\mathbf{Z}_A/\mathrm{R}_0$ is plotted. The lines $C\mathbf{Z}_{AN}$ and $M\mathbf{Z}_{AN}$ are drawn. The line $\mathbf{Z}_{1N}K$ is drawn from $\mathbf{Z}_{1N}$ so that $\angle M\mathbf{Z}_{AN}C=\angle M\mathbf{Z}_{1N}K=\phi$. The intersection of the lines $M\mathbf{Z}_{AN}$ and $K\mathbf{Z}_{1N}$ is $\mathbf{Z}_{LN}{}^*$.

The symmetrical network used for the graphical construction shown in Fig. 1 is the section of lossless transmission line shown in Fig. 2. The characteristic impedance of the line is 100 ohms, its length is 0.15 wavelength, $R_0=50$, $\mathbf{Z}_1=98.21+j70.05$, $\mathbf{Z}_L=25$ $j75$, and $\mathbf{Z}_A=17.03+j27.94$.

Test for Losses

Often four-terminal networks and two-port junctions are assumed to be lossless. This can be checked experimentally by terminating the network in at least two lossless loads, such as a short circuit and an open circuit. If two different input impedances are measured which are pure reactances, then the network is lossless.

Test for Symmetry

The term *symmetrical network* is used here in the sense that the electrical characteristics of the network are symmetrical at the frequency being used. It is not necessarily symmetrical in appearance or at other frequencies.

A convenient procedure for checking experimentally the symmetry of a lossless network is to terminate the network in a short circuit. Let $\mathbf{Z}_{SC}$ denote the corresponding input impedance. The point $\mathbf{Z}_{SCN}=\mathbf{Z}_{SC}/R_0$ is plotted. The network is symmetrical if and only if the point $\mathbf{Z}_{SCN}$ lies on the line from $\mathbf{Z}=0$ to M. Another test is that

$$X_{SC}=\frac{R_1{}^2+X_1{}^2-R_0R_1}{X_1}, \qquad (2)$$

where $\mathbf{Z}_{SC}=jX_{SC}$ and $\mathbf{Z}_1=R_1+jX_1$, if and only if the network is symmetrical.

For the network shown in Fig. 2, $\mathbf{Z}_{SC}=j137.64$ and $\mathbf{Z}_{SCN}$ lies on the line from $Z=0$ to M, as shown in Fig. 1. The network in Fig. 2 is made unsymmetrical by the addition of the shunt reactance $X_2=200$ as shown in Fig. 3 (next page). The value of $\mathbf{Z}_{SCN}$ remains unchanged and the new value of $\mathbf{Z}_1$ is $121.4+j84.44$. The point $\mathbf{Z}_{SCN}$ does not lie on the line from $\mathbf{Z}=0$ to M, as shown in Fig. 4. If $\mathbf{Z}_{SC}=0$, then $\mathbf{Z}_{1N}$ must lie on the circle shown in Fig. 5 when the network is symmetrical.

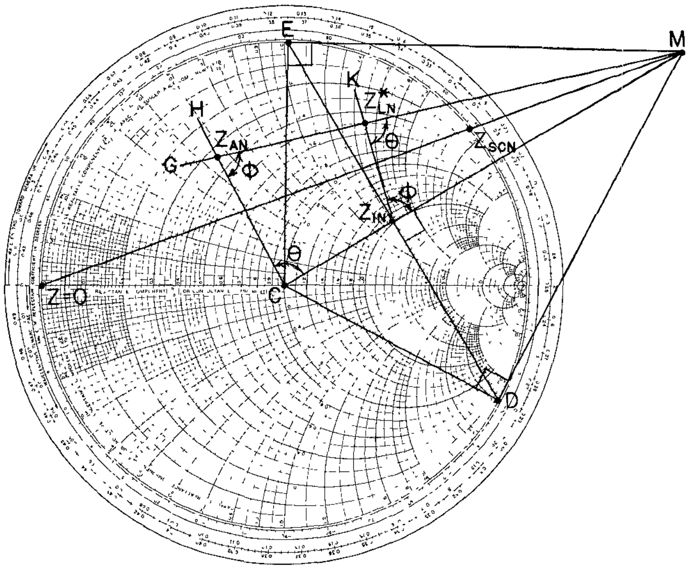

Fig. 1—Graphical construction for a symmetrical network.

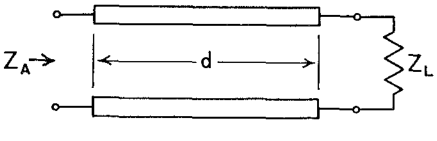

Fig. 2—A symmetrical network.

---

* Received January 14, 1963.

[1] D. Kajfez, "Wide-band matching of lossless waveguide two-ports," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 174–178; May, 1962.

Fig. 3—An unsymmetrical network.

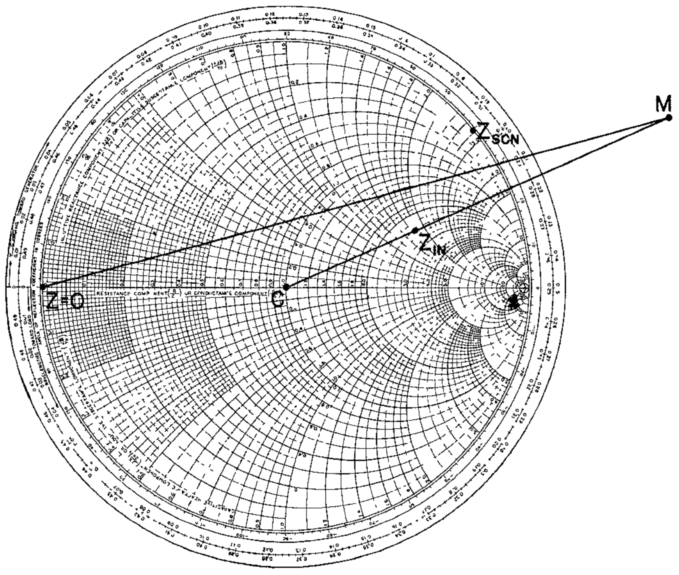

Fig. 4—Graphical test for symmetry.

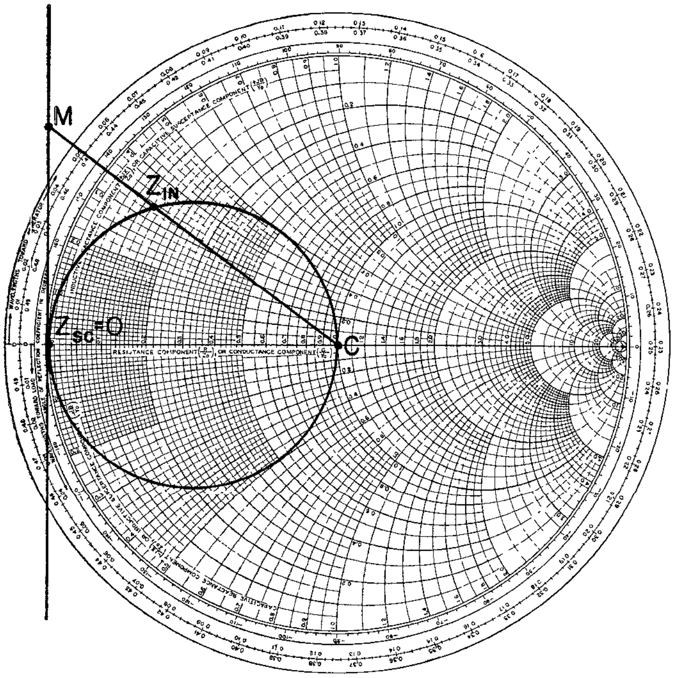

Fig. 5—Location of Z_{1N} for a symmetrical network with $Z_{SC}=0$.

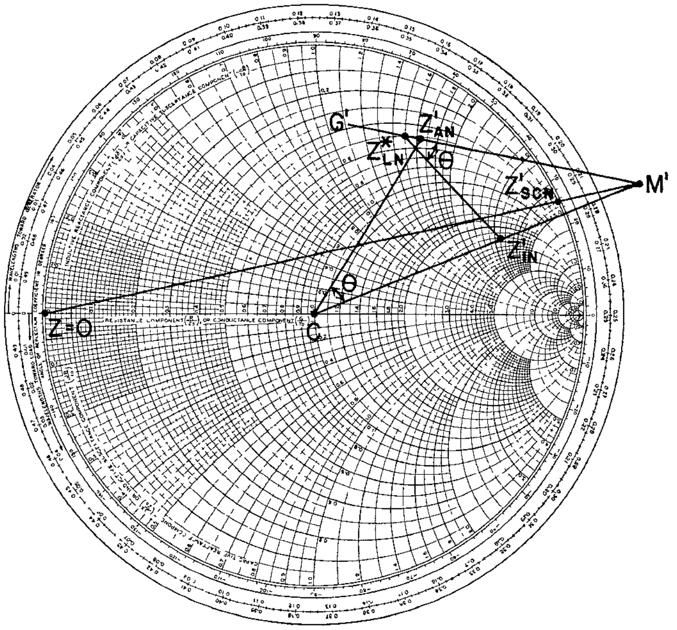

Fig. 6—Graphical construction for an unsymmetrical network.

Modification for Nonsymmetry

An unsymmetrical network can be made into a symmetrical network by simply adding the reactance

$$X_3 = \frac{X_1(X_{SC} - X_1) - R_1(R_1 - R_0)}{X_1 - X_{SC}} \tag{3}$$

in series at the input terminals, as illustrated in Fig. 3. For the circuit shown in Fig. 3, $X_3 = 78.53$. The new value of Z_{SC} is $Z_{SC}' = j216.17$, and Z_1 becomes $Z_1' = 121.41 + j162.96$. The same graphical construction is used as in Fig. 1, and it is shown in Fig. 6. Now $Z_A = R_0 Z_{AN}' - jX_3$. The reactance X_3 is simply added to Z_1 and subtracted from the value of Z_A which is obtained; it is not physically added to the circuit. Here, $Z_A = 25.89 + j3.23$.

Applications

There are many graphical and analytical techniques for transforming an impedance through a lossless four-terminal network or two-port junction. It is difficult to make any general statements about the advantages and disadvantages of any of these techniques. When these techniques are being compared, consideration should be given to the number of points to be plotted, lines to be drawn, angles to be measured, arithmetic operations to be performed, etc. Naturally, the saving of time is only important when one has many impedances to transform. The technique described here is especially useful when it is convenient to determine only Z_1 and Z_{SC}.

H. F. Mathis
Dept. of Elec. Engrg.
The Ohio State University
Columbus, Ohio